Mathematics teachers' professional development and identity in a distance education setting[1]

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This paper discusses the influence of an in-service distance education course in the construction of mathematics teachers' professional identity, especially regarding their views and practices of reflection and collaboration and their relation with information and communication technology. The course was based in open-learning pedagogy and focused on conducting exploratory and investigative work in the mathematics classroom. Evaluation results show that the perspectives and involvement of the participant teachers depend very much on their previous professional experience and relationship with the Internet. Teachers that use e-mail for collaborative work found this a very stimulating experience whereas those with less professional involvement had some difficulty in assuming the roles and values required for this kind of activity.

What influence might an in-service distance education course have on teachers' professional development and on the construction of their professional identity? This paper addresses this question based in a in-service course with the format of a "study circle", structured according to open-learning pedagogy (Collis & Moonen, 2001), and focused in conducting exploratory and investigative work in the mathematics classroom (Mason, 1991; Ponte, 2001; Ponte, Oliveira, & Brocardo, 2003; Ruthven, 2001; Skovsmose, 2000; Wood, 1994). An evaluation was undertaken to ascertain the reactions of the participants to the structure and functioning of this in-service activity and its influence in the professional development and identity of teachers. Special attention was given to how participants regard reflection and collaboration and relate to information and communication technology. Specifically, this paper discusses the main issues bearing on the emergence of new aspects of teachers' professional identity in this distance teacher education setting, and considers their potential implications for research and practice.

Teachers' professional development and identity

During their careers, mathematics teachers change and develop professionally. The professional development of a teacher may be regarded as a process of growing of his/her competencies concerning mathematics teaching practices and other professional practices, and in the self-control of his/her activity as an educator and as an active participant of the school organization. In this perspective, professional development concerns issues related to mathematics education but also includes issues related to every other aspect of the educational activity and personal as well as relational aspects regarding other teachers and members of the school, local, and educational communities.

Therefore, professional development is much more than acquiring bits and pieces of fragmented knowledge (Popkewitz, 1992). It includes learning more mathematics or

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mathematics education theories and getting able to use them in teaching practice but it is much more than that. In fact, professional development may be regarded as a complex process in which the teacher is involved as a person in his/her professional, cultural and social context and in close relationship with other teachers. It includes all the learning experiences of a teacher – natural and planned – that yield this teacher direct or indirect benefit that support the quality of his/her work with pupils and with other participants in educational institutions (Day, 1999).

For a mathematics teacher, the development of a professional identity involves assuming the fundamental roles, norms and values of the teaching profession. The key features of this identity are highly problematic since teachers are not a homogeneous body but encompass several professional subcultures (Feiman-Nemser & Floden, 1986). Key aspects of the culture of the mathematics teacher include the way he/she regards his/her professional role – as a transmitter of knowledge or a promoter of pupils' development – the norms of interaction with pupils and colleagues, the participation in professional activities such as projects, meetings and informal groups, and the teachers' own stance regarding his/her development. The way teachers relate to other teachers – working mostly individually or collaborating in important issues is an important indicator of their professional culture (Hargreaves, 1996).

Furthermore, professional identities have a dynamic nature, evolving along with the changes that occur in the society and in the nature of occupations. For example, in recent years, in several countries, the structure of the teachers' activity has undergone important changes involving new curriculum goals and professional responsibilities. Also, as it happens in many other fields, information and communication technology provides new opportunities to the teachers' work – using such technology in classes, preparing lessons and classroom materials, writing reports, carrying out administrative work, and sharing information and experiences with other teachers.

The professional identity is an aspect of the social identity. For Berger and Luckman (1973), in objective terms, a social identity can be regarded as belonging to a certain world, and can only be understood, in subjective terms, together with that world. The social identity is made of multiple aspects, including language, culture, social group, political and religious beliefs, and the role assumed in the social division of labour. The most important activity systems in the construction of the identity are communication that structures the interaction among individuals, and instrumental activity, related to the working processes and their underlying technical and organizational means (Dubar, 1997).

In the perspective of symbolic interaccionism (Blumer, 1969), the individual is not just a passive element of a group, that internalises its norms and values, but it is also an agent that assumes in that group a useful and recognized role. Therefore, we may speak of a dialectic between an "I" identified and recognized by the other as a member of the group and an "I" that assumes an active role and that participates in the permanent process of reconstruction of the community. For Dubar (1997), the harmonious integration of these two sides of "I" is the key to the consolidation of the social identity.

The social identity is permanently reconstituted in the socialization process. That is, the identity is not given, but continuously constructed and reconstructed in conditions of permanent uncertainly. So, social identities arise as dynamic entities and not as "objective data" or "subjective feelings". As Dubar states, "the social identity is not more than the result simultaneously stable and provisional, individual and collective, subjective and objective, biographic and structural, of the several socialization processes that, together,

individuals and institutions construct" (1997, p. 105). In his view, with the notion of social identity we can make a deeper analysis of professional issues than with the classical notions of group, class and category (of macrosociology) and role and status (of microsociology), as it "introduces a subjective, lived, psychic dimension in the kernel of sociological analysis" (p. 105).

Modern psychological and sociological theorists speak of decentring, displacement, or fragmentation of the subject, according to which individuals assume multiple identities (Bruner, 1990; Hall, 1999). This is a phenomenon that is increasingly occurring in our society – we have not one but several identities, sometimes contradictory and unsolved. The processes of identification, through which we project ourselves in our cultural identities has become more provisional, variable and problematic.

One of the factors that contribute to this change in the nature of social identities is information and communication technology, specially the Internet. This is a multisided new media that allows a wide range of personal experiences. First, it is an immense assembly of resources with information about events, news, documents, papers, lesson plans, *software,* and so on. Second, it also enables the publication of our own productions – papers, classroom plans, software, video clips, PowerPoint presentations, etc. – making them available to a wider public. But, third, more than an instrument of gathering information and disseminating educational products, the Internet provides the possibility of virtual interaction among people, including teachers, pupils, parents, future teachers, teacher educators, scientists, professionals, politicians, and many other social agents. The participants in these interactions form groups and networks that may be regarded as virtual communities, since their interactions are made through the cyberspace. The Internet constitutes, therefore, a new cognitive and social ecosystem in which individuals may embark in a process of adapting and restructuring their relational and cognitive activity, with possible consequences in the ways they view the world and they regard themselves. The extent to which this may happen and the specific features it may take are issues to be explored, notably as they affect mathematics teachers. In this paper, the main concern is how a distance education setting, based in a flexible pedagogy, stressing collaboration and reflection (both oral between pairs of teachers and written, writing papers and e-mail messages and contributing to a forum) assists the development of a teacher culture that puts a higher value on professional interactions and partnerships as well as on the use of information and communication technologies.

Methodology

Objectives, format, and participants

This paper examines a distance education in-service study circle that was attended by 36 teachers, 34 of whom completed it with success. A most prominent feature of this study circle was that teachers were required to register and work in pairs. The aim of this in-service activity was to offer teachers some theoretical ideas and practical experience regarding a current curriculum orientation – working with mathematical explorations and investigations – and to contribute towards their professional development, providing opportunities for reflecting on their own practice, using ICT, and developing a culture of collaboration. This course lasted for six months and it was divided in three segments: (i) dynamics of the mathematics classroom; (ii) investigations in mathematics and in

professional practice; (iii) one experiment with investigations in the classroom. The participants were mathematics teachers in grades 5 to 12 and come from different regions in Portugal (two from the rural North, eight from Porto; six from the Centre; thirteen from the Lisbon area, four from the city of Lisbon; one from Alentejo) and two are from Brazil.

The setting designed for this course includes a Web environment, through which various materials are provided. Teachers had to read mathematics education papers, conduct searches on the Internet and complete several tasks, some of which involved observation and reporting about their classrooms. For each segment, there is a study guide and several papers, some of which are required, others optional. These papers were to be read and discussed by each teacher with their partner and possibly with the teacher educator, and also with other participants in a mailing list. Some papers were written for this course and others were drawn from the professional and academic literature; all of them were in Portuguese (original versions or translations). Examples of required papers are Fonseca et al. (1999), Poincaré (1996), Ponte *et al.* (1997) and Skovsmose (2000).

The tasks were open and diversified. In task 1, the teachers had to comment on one of the required papers; in task 2, they had to describe and analyse a classroom situation that they had experienced; in task 3, they had to select and analyse a Web site relevant to mathematics investigations; in task 4, they had to study a problem from the history of mathematics, and, in task 5, they had to design a mathematical investigation, use it in their classroom, and to reflect on this experience. The open nature of these tasks enabled the participating teachers to carry them out according to their interests and concerns. The emphasis on writing, a multirepresentational and integrative process, favours the restructuring of meaning and constitutes, therefore a key activity in promoting reflection (Zabalza, 1994).

Dynamics and roles

This in-service course had three sections (two on Numbers/Functions and one on Geometry), organized according to the teachers' preferences. The first session was an introduction to the activities and working procedures and was carried out face to face at the university allowing participants and teacher educators to get to know each other. The last (double) session was a presentation and discussion by the participants of their work and a reflection about the study circle.

There were several different kinds of interactions among the participants:

- Teacher educators and teachers interact face to face (in the first and in last session,);
- Teachers interact with their partner teacher, as they work collaboratively;
- Teachers interact with the system, downloading materials and looking for information on the Web site and elsewhere;
- Teachers interact with teacher educators, via email and the Web site, sending tasks, answering questions, and reporting their progress;
- Teachers and teacher educators interact in a discussion list.

Besides the participating teachers and teacher educators, the course involved a coordinating team, overseeing the whole system, a technician who took care of the Web environment, and a team of external evaluators.

Regarding his/her participation in the study circle, each teacher was evaluated taking into account three main aspects: carrying out the tasks, participating in the discussion list, and self and group evaluations.

Program evaluation

The program also had an internal ongoing evaluation and an external evaluation. One focus of interest was the setting and the materials used; another focus, which is the main concern of this paper, was its effects on the participants. For the external evaluation, the data collection methods were a questionnaire, interviews, observation, and document analysis. As their final task, the participants had to respond to an open-response anonymous questionnaire with open response questions concerning the activity of the course and self-assessment. The documents provided in the course and the assignments produced by the participants were another source of data. The meetings of teacher educators were observed and the messages exchanged by the participants and teacher educators as well as the messages sent to the discussion list were also taken into account. To study in detail the experiences of six teachers who participated in the course, there were also in-depth case studies of three groups, based in face-to-face interviews. In these interviews teachers were asked to refect on their experiences throughout the course – readings, assignments, collaboration with the partner, exchanges with the teacher educators and other participants. The three groups of teachers were chosen so that there were (i) teachers with many years of experience as well as novice teachers, (ii) teachers from different regions of the country, and (iii) teachers with different levels of involvement in the course. In each case, data was analysed following a detailed schema of categories addressing the issues of interest concerning the activity and setting of the course and the experiences of the participants. This paper draws mostly on the results from the questionnaire and two of these case studies.

Results

General evaluation

As indicated by the responses to the questionnaire, the distanced education course was generally successful in attaining the set goals. Teachers become more aware of the issues involved in carrying out investigations in their mathematics classes, had opportunities to work closely and face to face with the colleagues with which they developed a close collaboration and to discuss at distance with their teacher educators and other course participants. Furthermore, in their responses most of the teachers (about 85%) found the structure of the course, the materials provided, and the tasks proposed useful and adequate for their professional development. These are some of the responses to the questionnaire that underline this:

> My commitment to the study circle was strong and fruitful. The discussions towards the preparation of each assignment were rich and involving.

> This distance format led my commitment in this study circle greater than in any other in-service course that I attended during the school year.

In the teachers' responses to the questionnaire, there are hints pointing to the development of new aspects of a professional identity. In particular, these concern the

attitudes of the participating teachers regarding reflection and regarding their relationship with information and communication technology. In fact, about half of the teachers indicate that, in one way or another, they had opportunities to reflect on their classroom practice and professional practice, as illustrated by the following responses:

> I reflected, I was questioned, I was commented, and I was evaluated about my views regarding mathematical investigations (…) The study circle represented a reflection about our own conscientiousness; we stoped to reflect about our professional life (…) I feel that the study circle led my to assume my own consciousness.

> We felt the need to reflect about our own practice. We had already carried out several investigation activities with our pupils (more or less guided) but we had not stoped to reflect in an organized way about such classes.

This last teacher talks directly about 'consciousness', which reflects a deep awareness of self and directly evokes the issue of identity. Only four teachers in the all group showed this kind of self-awareness. The first response speaks of reflecting in an organized way to improve practice. Carrying this out in a consistent way may also bring about in the long run changes in the teacher's identity.

The teachers also indicate several reasons that promoted such reflective stance. These included (i) working with a partner in team work, what, as one said, "yielded good opportunities to reflect about our activity"; (ii) the material provided, since "in the papers we also found hints to reflect"; and (iii) the questions posed by the teacher educators, as those "helped in the development of my reflecting and analysing ability". As a result of such reflection, two of the teachers indicated that they changed some of their conceptions regarding mathematics teaching:

> I was led to reflect about issues already addressed in my master's course (…) that I ended up seeing in a different way.

> I changed the meanings that I ascribed to several things that I had got from the *ProfMats* [national meetings of mathematics teachers], publications, NCTM Standards…

In their responses to the questionnaire, most of the teachers who had poor skills in using information and communication technology indicated that they developed them and others recognized that this was a useful way to get materials to use in their teaching practice:

> I learned how to use the Internet.

> [I got to know] a lot of information from the Internet (essential in this kind of work) that I was not aware before.

> I also learned that it is complicated to do searches on the Internet… Some perseverance is necessary to find good material.

Overall, these results point towards the development of a positive stance regarding reflection and collaboration, and also towards a better relationship or use of the Internet. Given the openended nature of the questions, these results just show a general trend. However, one could suspect that there was a wide variation of working experiences in this course. That is why case studies were conducted and we turn to them now.

Case 1

One of the groups included two teachers, Isaura and Anabela. They worked in different schools but had met a couple of years before, when Anabela was a student teacher and Isaura was her school supervisor. We may regard this as a heterogeneous group, as Isaura was much more experienced then Anabela. They were very interested in getting "practical" ideas, that they could take to their classrooms and regarded with little interest all that they consider "theoretical". They viewed the discussion list that was provided in the course as a place were theoretical discussions could be carried out and they had little participation in it. Isaura and Anabela showed interest in interrogating some aspects of their practice, notably those more closely related to the mathematical content to be taught. They carried out productive work during the course, although they had a rather difficult moment when the teacher educator indicated that they had to reformulate one assignment.

These two teachers enjoyed the fact that they could work in a very flexible way and did not have to displace themselves to attend meetings in a teacher education institution. However, in the interview, they indicate some uneasiness because of the absence of face-to-face interactions:

> At the beginning I missed the personal contact between us and the teacher educators and the other groups… (Anabela)

> This is a weakness. We are used to [direct] interpersonal relationships, to discuss face to face. And we are not used to do this at distance (…) It was difficult to be working and not to have immediate feedback from the other side. (Isaura)

The activity of the course required a lot of writing (messages to send to the teacher educator and to the discussion list and tasks that had to be delivered at certain times). However, these teachers did not seem comfortable in express themselves in writing. Isaura recognized it explicitly: "We find it difficult to express our opinion in front of a computer". The tone of their messages was formal, just saying the strictly necessary. This is in sharp contrast in the warm way they relate to other people in face-to-face settings as we witnessed in the meetings carried out at the university and in the interviews.

These two teachers see in a very positive way the fact that in this course they do not work in isolation but in pairs. In their view, working at a distance could become painful: "One person is completely isolated" (Isaura). In their perspective, pairs must be made up of people that like to work together: "I like if it is with someone that I like to work with" (Anabela). That is, they see working with a partner mostly as a motivation to overcome the difficult parts of the course.

These two teachers do not report major problems in dealing with information and communication technology. Isaura claimed to be experienced in using this technology, especially "to search things" in the Internet. Anabela recognized she had little experience, but indicated that in the study circle "she could do everything that was required". However, in the tasks proposed, these teachers had some trouble in interpreting what was asked from them. The teacher educators considered that their work did not correspond to the course's expectations and asked for reformulations. Isaura and Anabela felt quite uneasy with this demand and questioned themselves if they should drop the course. Assuming that the next tasks would be more interesting, they decided to continue. There were clearly difficulties in communication between these teachers and the teacher educators and different levels of expectation regarding the quality of the work to be carried out. The fact that all this negotiation had to be carried out by e-mail seemed to strengthen this difficulty.

In summary, these two teachers showed interest in questioning some aspects of their professional practice, notably those closely related to the topics that they had to teach. However, they showed little interest in analysing aspects related to the curriculum or the classroom dynamics, that they found as too "theoretical" and, therefore, with little relevance to their practice. They were able to use the information and communication technology to gather information and interact with the teacher educators but felt rather uneasy about this form of communication and reported to miss the face-to-face interactions of usual teacher education courses. That is, these two teachers represent a case of very little adherence to new forms of communication and professional interaction and therefore of developing their identity, as proposed in this teacher education setting.

Case 2

Before they were involved in this distance teacher education course, Julia and Maria, together with two other colleagues, already constituted an informal working group carrying out several professional activities. They used to meet face to face once a week and constantly exchanged e-mail messages among themselves (according to Maria, about ten daily messages…). They recognize that the participation in this course led them to write much more than they usually did. Julia, in the interview, stated that this was quite positive:

> There is an aspect (…) that I feel since a long time. Many times we have very interesting experiences and we discuss issues, and so on, and we always postpone writing, passing into the paper, recording. This distance education setting requires that we write.

At first, these teachers reacted negatively to the limit in the number of words that they could write in their assignments. But, with time, they recognized that such limitation led them to a deeper reflection about what they wrote: "That may be an aspect that created discipline in us, in the sense of reflecting again, improving, and fixing the essential things and also helping us to focus objectives that otherwise would remain somehow diluted" (Julia).

For Julia and Maria, writing does not seem to be a natural means of expression but they recognize its importance in their professional practice. As Julia said in the interview, their dynamic of collaborative work become stronger with this course, which they regard as making a significant contribution towards their professional development:

> Both as a pair and individually, we assumed our professional personality, and that is deeply gratifying (…) Working in group helps to share the competencies of each person and helps to overcome personal inhibitions because of the friendly and responsible commitment of each one with herself and her partner.

Julia and Maria show high interest in reflecting about their practice and in carrying out activities steaming from such reflection. They look for ideas arising from their experience, from the work of other teachers, and from professional and educational literature. They read the papers suggested in the course with interest and used them to analyse what went on in their classrooms.

These teachers recognize that this distance teacher education setting led them to write much more than they were used to. In the interview, Júlia comments that as positive:

> There is one aspect that may be quite particular but that I have felt for a long time [...] It is that we often have very interesting experiences and we discuss issues, and so on, and we always postpone writing, passing them onto paper, recording and all of us four do that a lot. The fact that this is a distance course [...] compels us into writing...

For them, writing does not appear to be a natural act, perhaps more because of the diversity of demands that they feel at every moment than because of lack of fluency in writing. However, they recognize the importance of writing for their professional practice.

They show great commitment in acting in their classrooms but have also high concern with what happens in their professional community, recognizing that it will take a long process so that it may grow in activity and in social and educational influence. As Julia stated in the interview:

> For us, communicating at distance (searching the Internet, communicating by e-mail, and so on) was already a routine activity. But it was interesting to experience in the study circle the creating of patterns of communication between us (…) and the teacher educator. (…) We felt that it is necessary and urgent to create among teachers a communication culture such as this, in which people feel free to participate, within rules of politeness, without hurting each other.

These teachers already had a strong professional experience and had previously developed many professional projects. They were used to working collaboratively and to making intensive use of the Internet. In this course, supported by the technological media, they were able to assume a productive division of tasks and had the experience of reflecting and writing about professional issues. As a result of this process, they report to become more confident about curriculum and professional issues and developed a stronger sense about their professional culture. Julia and Maria, who are able to take advantage of information and communication technology in a very flexible way, constitute indeed a very unique group, showing strong professional commitment.

Conclusion

Teachers involved in this study circle experienced a process of professional development in their knowledge regarding mathematical investigations and in their ability to carry such activities in the classroom. Many of them developed a more reflective stance and had a positive experience of professional collaboration. Some of them also developed their inclination to use information and communication technology for professional purposes. Such evolution may be regarded as pointing to changes in their professional identity.

The professional culture of teachers tends to be strongly individualistic (Feimen-Nemser, 1986; Hargreaves, 1996). However, as this experience has shown, information and communication technology, used in a formal way in distance education courses and used in an informal way in professional exchanges, may provide an opportunity for teachers to develop a dimension of virtual interactions with other teachers. In some cases, these interactions may become strong enough that we may speak of virtual communities. We may also have groups of teachers for which the face-to-face and virtual interactions are interrelated in a strong and fruitful way, as with the case of Julia and Maria reported in this paper.

Writing is not a natural activity for many mathematics teachers. That is also a feature of their professional identity. They think of themselves as at ease in using fluently numbers and symbols but not written language. However, this kind of communication was an important feature of this in-service activity. Teachers had to read mathematics education papers, had to turn in written assignments, had do communicate by e-mail with the teacher educators and had to participate in a (written) discussion list. For most of them, this was not a normal way of expressing themselves and some felt rather uneasy about it. But there is ample evidence that the emphasis in writing helped teachers to think about professional

issues in a deeper way and helped them to develop a more reflective stance. Julia and Maria even recognize that this form of expression may become an important feature of the professional culture of mathematics teachers.

The teachers who participated in this study circle were different in many regards. For example, Anabela was just learning how to use information and communication technology; Isaura could do searches in the Internet but did not use it for communication purposes; Julia and Maria were already quite experienced and intensive users. The distance education study circle appeared to help all these teachers to explore new ways of using this technology – downloading papers, exchanging documents, discussing issues by e-mail, and participating in discussion lists. Of course, a deeper involvement in communicating and interacting through electronic media is not a process that develops in just one step, much less just during a single in-service experience. However, the course format, stressing reflection on practice, connection to theoretical ideas, writing about professional issues, collaboration among teachers, and interactions of teachers and teacher educators put an emphasis on professional values that may be regarded as significant in developing new aspects of these mathematics teachers' professional identity.

These two pais of teachers – Anabela and Isaura, on one side, and Julia and Maria, on another – showed to have a rather different relationship with information and communication technologies but also, and more importantly, in their professional stance. Anabela and Isaura were interested in getting some more ideas for their classroom practice. Julia and Maria wanted to explore the possibilities of distance education for their professional development and wanted to reflect on classroom issues but also in the curriculum and on wider professional issues. The teacher education course tried to match the interests of each group, according to curriculum and professional orientations and concerns. In a course like this, it would have been unlikely to see striking changes in these teachers; however, we witness how several aspects of their professional identities relate and in some cases are fostered by this kind of virtual teacher education setting .

This experience shows the potential for teachers' development of open learning distance teacher education stressing collaboration and writing and opens up interesting issues for the design of in-service activities and resources. It also raises questions for further investigation regarding the tendencies and constrains of teachers changing professional identities supported by virtual learning communities. Other course formats can be designed, with similar or different features, and we can think of more informal ways of using the Internet to support the interactions, written reflections, collaborations and exchanges among teachers as well as interactions and collaboration involving teachers and teacher educators. It will be interesting to know what these media and these forms of work may bring to mathematics teachers who want to develop professionally and what that may mean in terms of developing new sides of their professional identities, perhaps in a stronger relationship with the written word and with a greater appreciation for reflecting and collaborating practices.

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